J. Fluid Mech. (2006), *vol.* 560, *pp.* 465–475.
doi:10.1017/S0022112006000140 Printed in the United Kingdom

Control of absolute instability by basic-flow modification in a parallel wake at low Reynolds number

By YONGYUN HWANG AND HAECHEON CHOI†

School of Mechanical and Aerospace Engineering, Seoul National University Seoul 151-744, Korea

(Received 29 September 2005 and in revised form 23 February 2006)

In this paper, we investigate the effect of basic-flow modification on the absolute instability in a two-dimensional wake at low Reynolds number with the parallel-flow approximation. Using the method of calculus of variation, we investigate how to modify the basic flow to suppress or enhance the absolute instability and suggest an optimal modification of the basic flow for stabilizing a bluff-body wake. In order to validate the present approach, we also measure the sensitivity of all the eigenvalues including the absolute-instability frequency, using the ϵ-pseudo-spectrum, showing that small modifications in the basic flow do not destabilize other eigenvalues by more than the original absolute-instability frequency, at least for the Reynolds number considered here. For a two-dimensional parallel model wake and a circular-cylinder wake, the present approach shows that the positive and negative velocity perturbations to the basic flows, respectively, at the wake centreline and separating shear layer suppress the absolute instability.

1. Introduction

The concept of absolute and convective instabilities originally developed from plasma physics (Briggs 1964) has provided important findings on the onset of vortex shedding in the wake behind a bluff body (Huerre & Monkewitz 1990; Chomaz 2005). In a bluff-body wake, the basic flow (i.e. steady unstable solution) undergoes changes in the nature of the local instability along the streamwise direction: the flow exhibits absolute instability in the near wake, whereas its nature is convectively unstable or linearly stable in the far wake.

The relation between this spatially developing instability property and the dynamics of global oscillation was subsequently investigated. In theoretical studies based on the WKBJ approximation (Chomaz, Huerre & Redekopp 1988, 1991; Monkewitz, Huerre & Chomaz 1993), it was shown that a finite region of local absolute instability is necessary for the onset of temporally growing linear instability, so-called linear global instability. In a bluff-body wake, this linear global instability is triggered when the Reynolds number exceeds a critical value (Zebib 1987; Jackson 1987). In this situation, the Hopf bifurcation occurs, and the instability wave is temporally amplified and eventually saturates to a limit-cycle oscillation, so-called vortex shedding. This

† Author to whom correspondence should be addressed: choi@snu.ac.kr. Also at National CRI Center for Turbulence and Flow Control Research, Institute of Advanced Machinery and Design, Seoul National University.

has been validated by experiments of Mathis, Provansal & Boyer (1984), Provansal, Mathis & Boyer (1987) and Schumm, Berger & Mankewitz (1994).

Recently, fully nonlinear aspects of vortex shedding have also been investigated by front propagation. In a wake, the velocity of the front determining the nonlinear characteristics of vortex shedding (e.g. threshold and frequency selection in the fully nonlinear sense) is determined by linear instability properties. Therefore, the local linear instability property of the basic flow is important even in the nonlinear dynamics of vortex shedding. For more details, refer to Chomaz (2005), which extensively reviews the recent progress on nonlinearity and non-normality of global instability.

The control of vortex shedding has been one of the central research issues in bluff-body wakes because vortex shedding has significant effects on the flow characteristics of such wakes. So far, many passive and active methods have been suggested for suppressing vortex shedding. Examples are base bleed (Wood 1964; Bearman 1967), base suction (Hammond & Redekopp 1997; Leu & Ho 2000), splitter plate (Roshko 1955; Kwon & Choi 1996), secondary cylinder (Strykowski & Sreenivasan 1990), periodic rotation of the cylinder (Tokumaru & Dimotakis 1991; Choi, Choi & Kang 2002; Protas & Wesfreid 2002; Protas & Styczek 2002), linear proportional control (Roussopoulos 1993; Park, Ladd & Hendricks 1994), suboptimal control (Min & Choi 1999), distributed forcing (Kim *et al.* 2004; Kim & Choi 2005), small-sized tab (Park *et al.* 2006) and so on. These successful control methods have been developed based on physical intuition and/or systematic control theory.

The success of some control methods has been explained in terms of the change in the absolute instability in the near wake. For example, base bleed eliminates or weakens the absolute instability in the near wake and eventually suppresses vortex shedding (Monkewitz 1988). In the case of base suction having sufficiently large amplitude, the effective region of absolute instability shrinks due to increased non-parallelism in the near wake, thus vortex shedding is stabilized (Hammond & Redekopp 1997; Leu & Ho 2000). The role of a small secondary cylinder placed in the near wake to suppress vortex shedding is also conjectured to be associated with a change in the absolute instability (Strykowski & Sreenivasan 1990; Schumm *et al.* 1994). These studies have shown that the control methods such as the base bleed, base suction and secondary cylinder modify the basic flow in the wake and these basic-flow modifications stabilize vortex shedding by weakening the absolute instability in the near wake.

To clearly understand the effect of basic-flow modification on the absolute instability, a more systematic approach based on mathematical control theory needs to be developed. Then, the inverse problem such as 'what kind of basic-flow modification weakens or enhances the absolute instability in a bluff-body wake' can be systematically solved. Therefore, in the present study, we examine the sensitivity of absolute-instability frequency with respect to basic-flow modification using the method of calculus of variation under the assumption of parallel flow, and derive the optimal basic-flow modification stabilizing bluff-body wake. Finally, we apply the present approach to a parallel model wake and a circular-cylinder wake, and show how basic-flow modifications change the absolute instability.

2. Sensitivity of the absolute-instability frequency to basic-flow modification

In order to calculate the first variation of absolute-instability frequency with respect to the modification of basic flow, we consider the following Orr–Sommerfeld equation that describes the dispersion relation of parallel flow:

$$-\mathrm{i}\omega \mathrm{M}\psi + \mathrm{L}_{os}\psi = 0, \qquad (2.1a)$$

with boundary conditions

$$\psi|_{\partial\Omega} = \mathscr{D}\psi|_{\partial\Omega} = 0, \tag{2.1b}$$

where

$$\mathrm{M} = \alpha^2 - \mathscr{D}^2, \tag{2.1c}$$

$$\mathrm{L}_{os} = \mathrm{i}\alpha U(\alpha^2 - \mathscr{D}^2) + \mathrm{i}\alpha\mathscr{D}^2 U + \frac{1}{Re}(\alpha^2 - \mathscr{D}^2)^2. \tag{2.1d}$$

Here, ψ is the stream function of velocity perturbation, α the streamwise wavenumber, ω the frequency, $U = U(y)$ the basic flow, $\mathscr{D} = \mathrm{d}/\mathrm{d}y$, Ω the flow domain in the transverse direction y, $\partial\Omega$ the boundary of Ω, and Re is the Reynolds number.

We consider small variations of the basic flow and streamwise wavenumber. Then these variations cause changes in the temporal frequency and eigenfunction of the Orr–Sommerfeld operator, i.e.

$$\begin{cases} U \to U + \epsilon\delta U \\ \alpha \to \alpha + \epsilon\delta\alpha \end{cases} \Rightarrow \begin{cases} \omega \to \omega + \epsilon\delta\omega \\ \psi \to \psi + \epsilon\delta\psi, \end{cases} \tag{2.2}$$

where $\epsilon \ll 1$. Using the zero group-velocity condition ($\partial\omega/\partial\alpha|_{\alpha=\alpha_0} = 0$) for the absolute instability, the first variation of absolute-instability frequency is given by

$$\delta\omega_0 = \int_\Omega K_0(y)\delta U(y)\,\mathrm{d}y, \tag{2.3a}$$

where

$$K_0(y) = \frac{\alpha_0\left[\alpha_0^2\psi_0(y)\overline{\phi}_0(y) + 2\mathscr{D}\psi_0(y)\mathscr{D}\overline{\phi}_0(y) + \psi_0(y)\mathscr{D}^2\overline{\phi}_0(y)\right]}{\displaystyle\int_\Omega \overline{\phi}_0\mathrm{M}\psi_0\,\mathrm{d}y}. \tag{2.3b}$$

Here, an overbar denotes the complex conjugate, α_0 is the absolute-instability wavenumber, ω_0 the absolute-instability frequency, and $\psi_0(y)$ and $\phi_0(y)$ are the corresponding regular and adjoint eigenfunctions, respectively. The adjoint eigenfunction is obtained by solving the adjoint equation corresponding to (2.1). $K_0(y)$ is the sensitivity of absolute-instability frequency with respect to the modification of the basic flow. The same formula based on the classical linear stability problem was also obtained in Bottaro, Corbett & Luchini (2003), where, given the streamwise wavenumber, the sensitivity of the temporal frequency to the modification of the basic Couette flow was discussed.

As is well known from previous studies (Reddy & Henningson 1993; Trefethen *et al.* 1993; Schmid & Henningson 2001), the eigenvalues of the Orr–Sommerfeld operator are extremely sensitive to the perturbation of the operator because the operator is non-normal. Thus, (2.3) may not be applicable to the control problem, because other eigenvalues may be destabilized by small changes in the basic flow by more than the original absolute-instability frequency. However, we will show that the non-normality of the Orr–Sommerfeld operator is sufficiently moderate in a parallel wake at low Reynolds number (see § 4.1). Therefore, the imaginary part of $K_0(y)$, $K_{0_i}(y)$, plays a critical role in controlling the absolute instability. From the information on $K_{0_i}(y)$ one can determine $\delta U(y)$, which stabilizes or destabilizes the flow.

3. Optimal modification of a parallel basic flow

The growth rate of the absolute instability frequency is the critical parameter for controlling the absolute instability. Equation (2.3) shows how the absolute-instability frequency ω_0 changes due to the modification of the parallel basic flow δU. Thus, let us consider the following optimization problem:

$$\min_{\delta U} \delta\omega_{0i} \quad \text{subject to} \quad \int_\Omega \delta U^2(y)\,\mathrm{d}y = c, \tag{3.1a}$$

where

$$\delta\omega_{0i} = \int_\Omega K_{0_i}(y)\delta U(y)\,\mathrm{d}y. \tag{3.1b}$$

Here, $0 < c \ll 1$. In (3.1*a*) the constraint represents the condition for a fixed amount of basic-flow modification. Using the Lagrange multiplier, the optimal δU in stabilizing the absolute instability ($\delta\omega_{0i} < 0$) is obtained as

$$\delta U(y) = -c\frac{K_{0_i}(y)}{\sqrt{\int_\Omega K_{0_i}^2(y)\,\mathrm{d}y}}. \tag{3.2}$$

4. Parallel model wake

In this section, we discuss the sensitivity of eigenvalues of the Orr–Sommerfeld operator for a parallel model wake at low Reynolds number using the formulae derived in the previous sections.

Following Monkewitz (1988), the profile of the basic flow as a parallel model wake is given as

$$U(y) = 1 - \Lambda + 2\Lambda F(y), \tag{4.1a}$$

where

$$\Lambda = (U_c^* - U_\infty^*)/(U_c^* + U_\infty^*), \tag{4.1b}$$

$$F(y) = [1 + \sinh^{2a}\{y \sinh^{-1}(1)\}]^{-1}. \tag{4.1c}$$

Here, the superscript * denotes a dimensional quantity, $U_c^*(= U^*|_{y=0})$ is the centreline velocity, and $U_\infty^*(= U^*|_{y=\infty})$ is the free-stream velocity. The Reynolds number is defined as $Re_b = \widehat{U}^* b/\nu$, where $\widehat{U}^*(= (U_c^* + U_\infty^*)/2)$ is the average basic-flow velocity, b is the wake half-width such that $U^*|_{y=b} = \widehat{U}^*$, and ν is the kinematic viscosity. In this study, $\Lambda = -1.105$, $a = 1.34$ and $Re_b = 12.5$ are chosen following Monkewitz (1988). The corresponding velocity profile is shown in figure 1(*a*). The velocity is slightly negative for $-0.377 \leqslant y \leqslant 0.377$ and positive elsewhere.

The regular and adjoint Orr–Sommerfeld equations are solved using the standard Chebyshev collocation technique (Canuto *et al.* 1988) with $N = 100$ to accurately resolve all significant eigenvalues. The Chebyshev–Gauss–Lobatto points, $-1 \leqslant \zeta_j = \cos[(j-1)\pi/(N-1)] \leqslant 1$ for $j = 1, 2, \ldots, N$, are mapped onto the transverse direction $-63 \leqslant y \leqslant 63$ through the cotangent mapping. The resulting matrix eigenvalue problem is solved using the subroutine `zggev.f` in the LAPACK library. The absolute-instability wavenumber α_0 and frequency ω_0 are obtained using the cusp map procedure described in Kupfer, Bers & Ram (1987): $\alpha_0 = 0.8075 - 0.4890\mathrm{i}$ and $\omega_0 = 0.9577 + 0.0628\mathrm{i}$. These absolute-instability wavenumber and frequency values are in good agreement with those in Monkewitz (1988) ($\alpha_0 = 0.831 - 0.505\mathrm{i}$ and $\omega_0 = 0.990 + 0.061\mathrm{i}$). Since ω_{0i} is positive, this parallel model wake is absolutely unstable.

4.1. *ϵ-pseudo-spectrum for a parallel model wake*

In this section, we show that the non-normality of a Orr–Sommerfeld operator is sufficiently moderate in parallel wake at low Reynolds number. Moderate

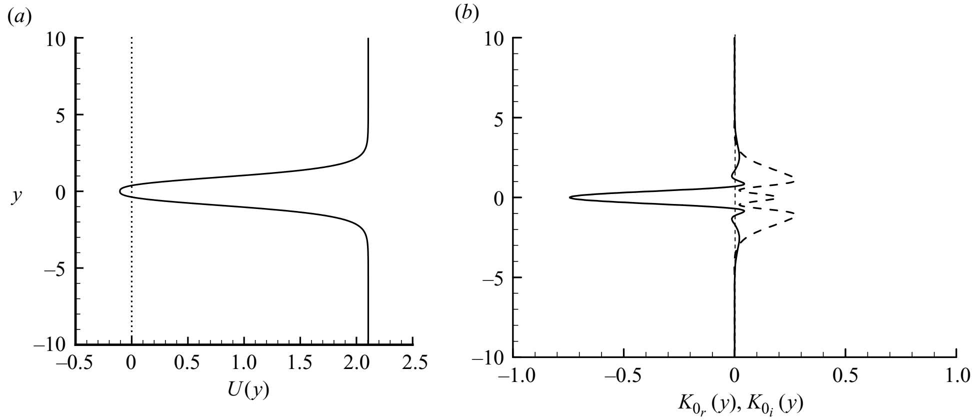

FIGURE 1. Parallel model wake U and sensitivity function K_0 ($\Lambda = -1.105$, $a = 1.34$ and $Re_b = 12.5$): (*a*) $U(y)$; (*b*) - - - -, $K_{0_r}(y)$; ——, $K_{0_i}(y)$.

non-normality indicates that small changes in the basic flow that stabilize the absolute-instability frequency do not destabilize other eigenvalues by more than the original absolute-instability frequency.

Let us first define the ϵ-pseudo-spectrum (or ϵ-pseudo-eigenvalue). The concept of ϵ-pseudo-spectrum has been used to qualitatively measure the sensitivity of eigenvalues of the Orr–Sommerfeld operator (Reddy & Henningson 1993; Trefethen *et al.* 1993; Schmid & Henningson 2001). Consider a linear time-invariant system $\partial\psi/\partial t = \mathrm{L}\psi$. A number z is an ϵ-pseudo-spectrum of L, if any of the following two conditions is satisfied: (i) z is an eigenvalue of $\mathrm{L} + P$ for some random perturbation matrix P with $\|P\|_E \leqslant \epsilon$; (ii) z satisfies $\|(zI - \mathrm{L})^{-1}\|_E \geqslant \epsilon^{-1}$. Here $\epsilon > 0$. In these definitions, the norm $\|\cdot\|_E$ of the linear operator is based on the following norm for an arbitrary square integrable function ψ in Ω:

$$\|\psi\|_E = \left[\int_\Omega (|\mathscr{D}\psi|^2 + |\alpha|^2|\psi|^2)\,\mathrm{d}y\right]^{1/2}. \tag{4.2}$$

From the definition (4.2), the norm of an arbitrary linear operator A is defined as

$$\|A\|_E \equiv \sup_{\psi\neq 0} \frac{\|A\psi\|_E}{\|\psi\|_E}. \tag{4.3}$$

These norms are called the energy norm.

In order to obtain numerical solutions of the ϵ-pseudo-spectrum, the Orr–Sommerfeld operator, $\mathrm{L} = -\mathrm{M}^{-1}\mathrm{L}_{os}$ (see (2.1)), is discretized using the Chebyshev collocation method with $N = 100$ as before. For accurate evaluation of the energy norm in (4.2), we use 3000 uniform grids in the transverse domain (y), and the ψ on these grid points are obtained through the Chebyshev transformation of ψ previously obtained with $N = 100$ and its inverse. Then, the integration and differentiation of ψ in (4.2) are performed on the uniform grids using the rectangle rule and second-order central difference method, respectively. After obtaining $\|\psi\|_E$, the energy norms, $\|P\|_E$ and $\|(zI - \mathrm{L})^{-1}\|_E$, are obtained using the singular value decomposition.

Figure 2 shows the eigenspectra and ϵ-pseudo-spectra for the model wake shown in figure 1(*a*) at $Re_b = 12.5$ and $Re_b = 100$. Similarly to the Poiseuille and Couette flows, there are three types of branch, denoted by A, P and S (see figure 2*a*, *e*)

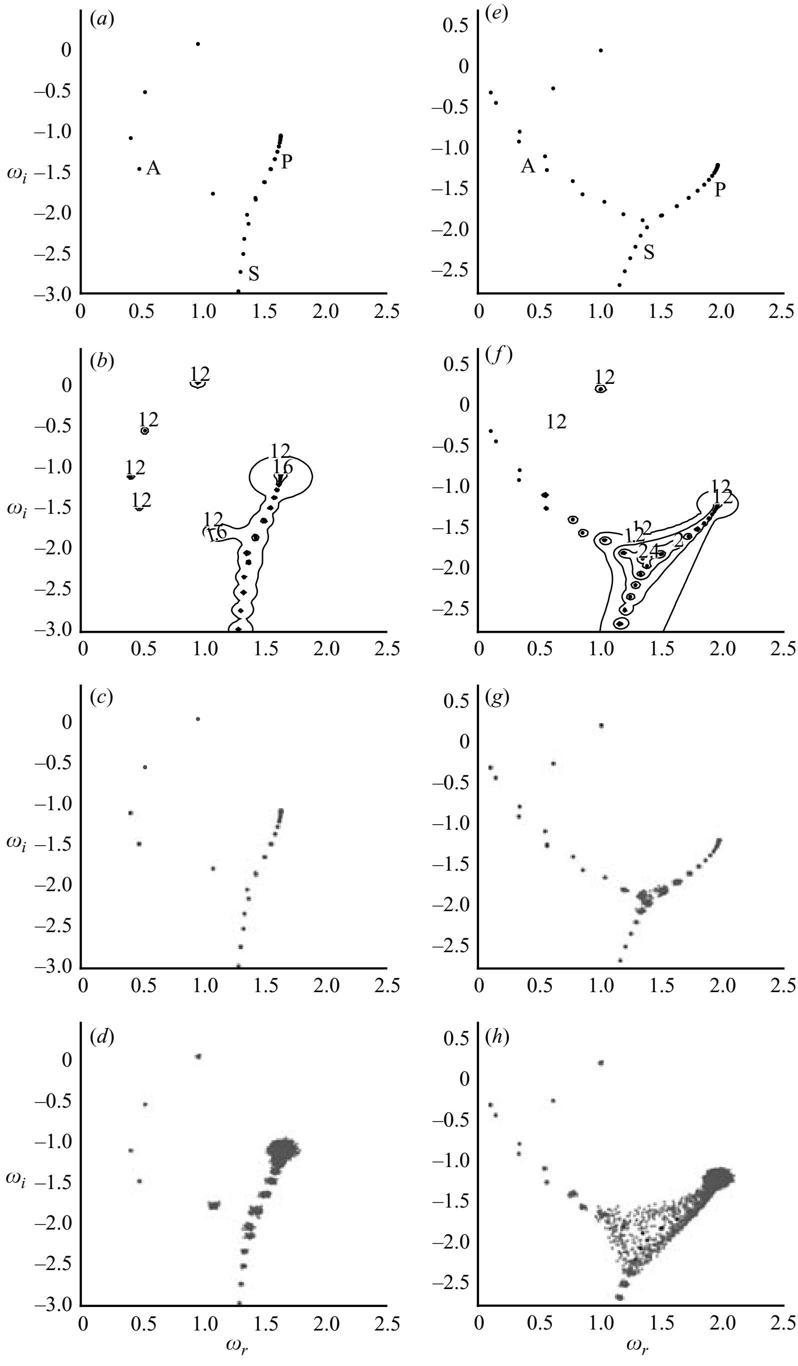

FIGURE 2. Eigenspectra and ϵ-pseudo-spectra for the parallel model wake ($\Lambda = -1.105$ and $a = 1.34$) for (*a–d*) $\alpha = \alpha_0$ ($= 0.8075 - 0.4890i$) and $Re_b = 12.5$, and (*e–h*) $\alpha = \alpha_0$ ($= 0.9375 - 0.5797i$) and $Re_b = 100$: (*a*, *e*) eigenspectra; (*b*, *f*) ϵ-pseudo-spectra based on the resolvent norm (contour levels represent $\log_{10} \|(zI - L)^{-1}\|_E$); (*c*, *g*) ϵ-pseudo-spectra based on the random perturbation of the operator ($\|P\|_E = 0.1$); (*d*, *h*) ϵ-pseudo-spectra based on the random perturbation of the operator ($\|P\|_E = 1$). In (*b*), (*d*), (*f*) and (*h*), eigenspectra are also plotted for comparison.

for this model wake. P and S branches form continuous spectra like the Blasius boundary layer (see Schmid & Henningson 2001). The ϵ-pseudo-spectra based on the resolvent norm (i.e. $\|(zI - \mathrm{L})^{-1}\|_E$) (figure 2$b$,$f$) and the random perturbation of the operator L (figure 2c, d) and g, h) are obtained by computing the resolvent norm and superposing eigenvalues from 50 different perturbed Orr–Sommerfeld operators on the (ω_r, ω_i)-plane, respectively. In case of the ϵ-pseudo-spectrum based on the resolvent norm, the regions having large $\|(zI - \mathrm{L})^{-1}\|_E$ are located at the upper edge of branch P and the region intersected by all three branches (figure 2b, f), indicating that the most sensitive eigenspectra are located at these regions. Significant modifications of eigenspectra from perturbation of the operator L also occur in these regions (figure 2c, d and g, h), showing that two different measures of the ϵ-pseudo-spectrum provide very similar results. The sensitivity of the intersection region is similar to those for the Poiseuille and Couette flows at $Re = O(10^3–10^4)$. However, in Poiseuille and Couette flows, the eigenspectra are completely modified by an operator perturbation of magnitude only $O(10^{-4}–10^{-5})$. On the other hand, the eigenspectra for the present parallel model wake at $Re_b = 12.5$ and $Re_b = 100$ are almost insensitive to an operator perturbation of magnitude even $O(10^{-1})$ (figure 2c, g). For an operator perturbation of $O(1)$, the eigenspectra are significantly modified at the branch P and the intersection region, but these modified eigenspectra are still stable compared to the absolute-instability frequency (figure 2d, h). Furthermore, these sensitive regions are located sufficiently far from the absolute-instability frequency ($\omega_0 = 0.9577 + 0.0628\mathrm{i}$ at $Re_b = 12.5$ and $\omega_0 = 1.0056 + 0.1835\mathrm{i}$ at $Re_b = 100$).

Therefore, small changes in the Orr–Sommerfeld operator do not destabilize other eigenvalues by more than the original absolute-instability frequency. This eigenspectrum characteristic is essentially caused by the low Reynolds number considered here. At low Reynolds number, the Orr–Sommerfeld operator is close to a normal operator due to the self-adjoint diffusion term, and this self-adjoint diffusion term causes the eigenspectra of the Orr–Sommerfeld operator to be almost insensitive to small perturbations.

4.2. *Application to a parallel model wake*

Now, we apply the formulae derived in §§2 and 3 to the present parallel model wake at low Reynolds number. With the solutions of $\psi_0(y)$, $\phi_0(y)$ and α_0, the sensitivity function $K_0(y)$ is obtained from (2.3b) and is shown in figure 1(b). K_{0_i} is positive in the separating shear layer ($0.7 \leqslant y \leqslant 1.15$ and $y \geqslant 1.7$) and negative near the centreline ($-0.7 \leqslant y \leqslant 0.7$). Note that K_{0_i} is slightly negative for $1.15 < y < 1.7$. We also considered other Λ and a values, but the resulting $K_0(y)$ were qualitatively similar to that shown in figure 1(b).

As described in (3.2), $K_{0_i}(y)$ determines the optimal δU for stabilizing the absolute instability. For example, a decrease in the basic flow ($\delta U < 0$) in the separating shear layer stabilizes the flow, whereas an increase in the basic flow ($\delta U > 0$) along the centreline in the recirculating region suppresses the absolute instability.

5. Circular-cylinder wake at $Re_D = 48$

In this section, we apply the present formulae to a circular-cylinder wake at $Re_D = u_\infty D/\nu = 48$, where u_∞ is the free-stream velocity and D is the cylinder diameter. To obtain the basic flow, we solve the Navier–Stokes equations numerically on a staggered Cartesian mesh using the immersed boundary method (Kim *et al.* 2001). The computational domain is $-50 \leqslant x \leqslant 70$ in the streamwise direction and $0 \leqslant y \leqslant 30$

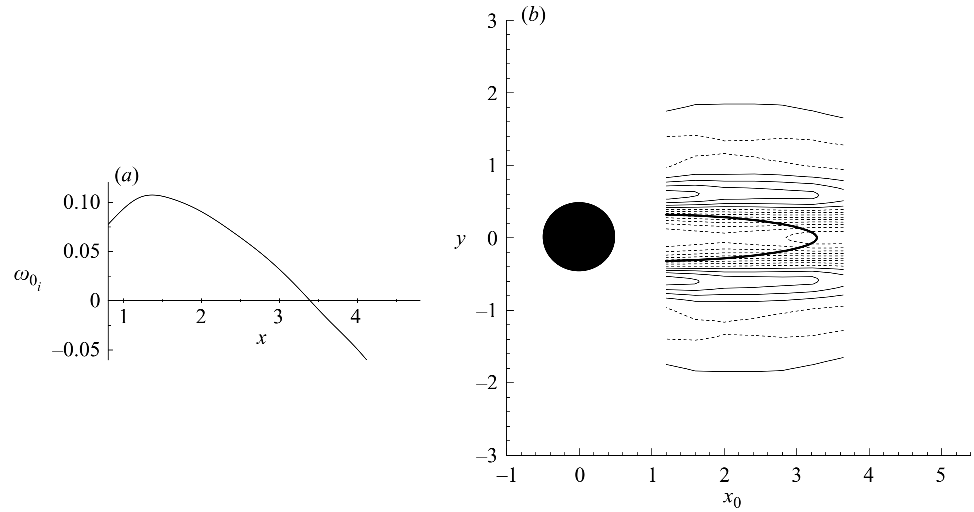

FIGURE 3. Distributions of the local absolute-instability frequency and sensitivity function in the streamwise direction (circular-cylinder wake at $Re_D = 48$): (*a*) $\omega_{0_i}(x)$; (*b*) $K_{0_i}(y; x_0)$. In (*b*), the solid and dashed lines denote, respectively, positive and negative values (-1.8 to 0.6 in increments of 0.2), and the thick solid line denotes the contour of $U = 0$.

in the transverse direction. Here, only the upper half-domain is considered and the symmetric boundary condition ($v = 0$ and $\partial u/\partial y = 0$) is imposed at $y = 0$, where u and v are the velocity components in the x- and y-directions, respectively. The number of grid points used is $641(x) \times 2048(y)$. The large number of grid points in the transverse direction is to capture a spectral-like accuracy of the Orr–Sommerfeld-equation solution using the Chebyshev collocation method. For more numerical details, refer to Kim *et al.* (2001). The properties of the computed basic flow such as the recirculation length ($L = 3.26D$ from the centre of the cylinder) and drag coefficient of the cylinder ($C_D = 1.41$) show good agreements with the results of Fornberg (1980, 1985).

To calculate local absolute-instability frequencies, the same numerical methods as in § 4 are used for solving the regular and adjoint Orr–Sommerfeld equations. The streamwise velocity at each x-location in the wake is provided from the simulation of the Navier–Stokes equations and the corresponding local absolute-instability frequency $\omega_0(x)$ is obtained. Figure 3(*a*) shows the imaginary part of the local absolute-instability frequency in the streamwise direction. This result is in good agreement with that of Pier (2002). Note that the flow is locally absolutely unstable in the near wake ($x \leqslant 3.39D$) at $Re_D = 48$. This region nearly coincides with the flow-reversal region.

The sensitivity function at each x_0 location, $K_0(y; x_0)$, is obtained using (2.3*b*) and its imaginary part is shown in figure 3(*b*), together with the contour of $U = 0$. The optimal basic-flow modification for stabilizing the absolute instability ($\delta\omega_{0_i} < 0$) is $\delta U(y) \propto -K_{0_i}(y)$ (see (3.2)). Therefore, to stabilize the vortex shedding at $Re_D = 48$, one has to decrease and increase the basic flow, respectively, in the separating shear layer and near the centreline. For example, positioning a secondary cylinder in the separating shear layer (or at the centreline) (Strykowski & Sreenivasan 1990) results in $\delta U < 0$ (or $\delta U > 0$) there and thus the flow becomes stabilized because $K_{0_i} > 0$ (or $K_{0_i} < 0$, respectively) there. Recently, Giannetti & Luchini (2003) studied the effect of basic-flow modification on the linear global instability in a circular-cylinder wake and showed that the region of separating shear layer is the most receptive to the basic-flow modification. On the other hand, base bleed (Wood 1964; Bearman 1967; Schumm *et al.* 1994) increases U there (i.e. $\delta U > 0$), and thus suppresses the absolute instability because $K_{0_i}(y) < 0$ along the centreline. Monkewitz (1988) conducted a parametric study on the absolute instability for base bleed, in which the variation of absolute-instability frequency was obtained by changing Λ and a in (4.1), and showed that base bleed suppresses the absolute instability. Although the exact amounts of basic-flow modification due to the secondary cylinder and base bleed are not evaluated here, the present result shows a qualitative agreement with those of previous studies. This clearly indicates that the stabilization of the global mode is closely related to the suppression of local absolute instability.

6. Concluding remarks

The concept of absolute and convective instabilities has played an important role in interpreting the dynamical behaviour of open shear flows (Chomaz 2005). The nature of local instability reflecting the dynamical characteristics of basic flow is closely associated with the global instability. In this respect, investigation of the relation between the basic flow and absolute instability should be useful for understanding the nature and control of open shear flows.

In the present study, we studied the effect of basic-flow modification on the absolute instability in two-dimensional parallel wakes at low Reynolds numbers. The sensitivity analysis of absolute instability suggested how one can modify the basic flow to suppress or enhance the absolute instability. For example, in a two-dimensional model wake and a circular-cylinder wake exhibiting an absolutely unstable nature, we showed that positive and negative velocity perturbations to the basic flow, respectively, at the centreline and separating shear layer suppress the absolute instability. We also showed that, using the ϵ-pseudo-spectrum, the non-normality of the Orr–Sommerfeld operator is sufficiently moderate for a two-dimensional parallel model wake at low Reynolds number. Thus, other eigenvalues were not destabilized by more than the original absolute-instability frequency.

As stated in a recent review by Chomaz (2005), the local stability analysis for linear global mode may not be applicable when the inherent linearized operator is strongly non-normal. However, the local stability analysis has provided a good framework for studying the linear global mode in many open shear flows such as wakes, backward facing steps, separation bubbles and so on. This indicates that the linearized operators near the onset of the global mode in these open shear flows may not be strongly non-normal, like the present wake at low Reynolds number. Chomaz (2005) also conjectured that the wake behind a circular cylinder has this property. Therefore, the present approach is applicable to these flows and provides a simple and useful framework for studying the nature of absolute instability.

This work is sponsored by the Creative Research Initiatives Program through the Korean Ministry of Science and Technology.

REFERENCES

Bearman, P. W. 1967 The effect of base bleed on the flow behind a two-dimensional model with a blunt trailing edge. *Aero. Q.* **18**, 207.

Bottaro, A., Corbett, P. & Luchini, P. 2003 The effect of base flow variation on flow stability. *J. Fluid Mech.* **476**, 293.

Briggs, R. J. 1964 *Electron-Stream Interaction with Plasmas*. MIT Press.

Canuto, C., Hussaini, M. Y., Quarteroni, A. & Zang, T. A. 1988 *Spectral Methods in Fluid Dynamics*. Springer.

Choi, S., Choi, H. & Kang, S. 2002 Charateristics of flow over a rotationally oscillating cylinder at low Reynolds number. *Phys. Fluids* **14**, 2767.

Chomaz, J. M. 2005 Global instabilities in spatially developing flows: non-normality and nonlinearity. *Annu. Rev. Fluid. Mech.* **37**, 357.

Chomaz, J. M., Huerre, P. & Redekopp, L. G. 1988 Bifurcation to local and global modes in spatially developing flows. *Phys. Rev. Lett.* **60**, 25.

Chomaz, J. M., Huerre, P. & Redekopp, L. G. 1991 A frequency selection criterion in spatially developing flows. *Stud. Appl. Maths* **84**, 119.

Fornberg, B. 1980 A numerical study of steady viscous flow past a circular cylinder. *J. Fluid. Mech.* **98**, 819.

Fornberg, B. 1985 Steady viscous flow past a circular cylinder up to Reynolds number 600. *J. Comput. Phys.* **61**, 297.

Giannetti, F. & Luchini, P. 2003 Recptivity of the circular cylinder's first instability. *Proc. 5th Eur. Fluid Mech. Conf., Toulouse*.

Hammond, D. & Redekopp, L. 1997 Global dynamics of symmetric and asymmetric wakes. *J. Fluid Mech.* **331**, 231.

Huerre, P. & Monkewitz, P. A. 1990 Local and global instabilities in spatially developing flows. *Annu. Rev. Fluid. Mech.* **22**, 473.

Jackson, C. P. 1987 A finite-element study of the onset of vortex shedding in flow past variously shaped bodies. *J. Fluid Mech.* **182**, 23.

Kim, J. & Choi, H. 2005 Distributed forcing of flow over a circular cylinder. *Phys. Fluids* **17**, 033103.

Kim, J., Hahn, S., Kim, J., Lee, D., Choi, J., Jeon, W.-P. & Choi, H. 2004 Active control of turbulent flow over a model vehicle for drag reduction. *J. Turbulence* **5**, 19.

Kim, J., Kim, J. & Choi, H. 2001 An immersed boundary finite volume method for simulation of flow in complex geometries. *J. Comput. Phys.* **171**, 132.

Kupfer, K., Bers, A. & Ram, A. K. 1987 The cusp map in the complex frequency plane for absolute instabilities. *Phys. Fluids* **30**, 175.

Kwon, K. & Choi, H. 1996 Control of laminar vortex shedding behind a circular cylinder using splitter plates. *Phys. Fluids* **8**, 479.

Leu, T. S. & Ho, C. M. 2000 Control of global instability in non-parallel wake. *J. Fluid Mech.* **404**, 345.

Mathis, M., Provansal, M. & Boyer, L. 1984 Bénard-von Kármán instability: an experimental study near the threshold. *J. Physique Lett.* **45**, 483.

Min, C. & Choi, H. 1999 Suboptimal feedback control of vortex shedding at low Reynolds numbers. *J. Fluid Mech.* **401**, 123.

Monkewitz, P. A. 1988 The absolute and convective nature of instability in two dimensional wake at low Reynods number. *Phys. Fluids* **31**, 999.

Monkewitz, P. A., Huerre, P. & Chomaz, J. M. 1993 Global linear stability analysis of weakly non-parallel shear flows. *J. Fluid Mech.* **251**, 1.

Park, D. S., Ladd, D. M. & Hendricks, E. W. 1994 Feedback control of von Karman vortex shedding behind a circular cylinder at low Reynods number. *Phys. Fluids* **6**, 2390.

Park, H., Lee, D., Jeon, W.-P., Hahn, S., Kim, J., Kim, J., Choi, J. & Choi H. 2006 Drag reduction in flow over a two-dimensional bluff body with a blunt trailing edge using a new passive device. *J. Fluid Mech.* (in press).

Pier, B. 2002 On the frequency selection of finite-amplitude vortex shedding in the cylinder wake. *J. Fluid Mech.* **458**, 407.

Protas, B. & Styczek, A. 2002 Optimal rotary control of the cylinder wake in the laminar regime. *Phys. Fluids* **14**, 2073.

Protas, B. & Wesfreid, J. E. 2002 Drag force in the open-loop control of the cylinder wake in the laminar regime. *Phys. Fluids* **14**, 810.

Provansal, M., Mathis M. & Boyer, L. 1987 Bénard-von Kármán instability: transient and forecd regimes. *J. Fluid Mech.* **182**, 1.

Reddy, S. C. & Henningson, D. S. 1993 Pseudospectra of the Orr–Sommerfeld operator. *SIAM J. Appl. Maths* **53**, 15.

Roshko, A. 1955 On the wake and drag of bluff bodies. *J. Aerospace Sci.* **22**, 124.

Roussopoulos, K. 1993 Feedback control of vortex shedding in the cylinder wake. *J. Fluid Mech.* **248**, 267.

Schmid, P. J. & Henningson, D. S. 2001 *Stability and Transition in Shear Flows*. Springer.

Schumm, M., Berger, E. & Monkewitz, P. A. 1994 Self-excited oscillations in the wake of two-dimensional bluff bodies and their control. *J. Fluid Mech.* **271**, 17.

Strykowski, P. J. & Sreenivasan, K. R. 1990 On the formation and suppression of vortex 'shedding' at low Reynolds numbers. *J. Fluid Mech.* **218**, 71.

Tokumaru, P. T. & Dimotakis, P. E. 1991 Rotary oscillation control of a cylinder wake. *J. Fluid Mech.* **224**, 77.

Trefethen, L. N., Trefethen, A. E., Reddy, S. C. & Driscoll, T. A. 1993 Hydrodynamic stability without eigenvalues. *Science* **261**, 578.

Wood, C. J. 1964 The effect of base bleed on a periodic wake. *J. R. Aero. Soc.* **68**, 477.

Zebib, A. 1987 Stability of viscous flow past a circular cylinder. *J. Engng Maths* **21**, 155.