Sensitivity of global instability of spatially developing flow in weakly and fully nonlinear regimes

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In the present study, we investigate the sensitivity of global instability in weakly and fully nonlinear regimes using the supercritical Ginzburg–Landau equation. In the weakly nonlinear regime, the sensitivity of global instability is determined by linear global mode and its adjoint mode, and the variation of sensitivity with the departure from the criticality does not change. On the other hand, it is shown that in the fully nonlinear regime, the sensitivity of global instability is characterized by nonlinear global mode and adjoint mode of secondary-instability equation, and the Kolmogorov front acts as a wavemaker of nonlinear global mode. The sensitive region of nonlinear global mode in the fully nonlinear regime is similar to that of linear global mode in the weakly nonlinear regime, but this Kolmogorov front layer becomes less sensitive to the perturbation as the departure from the criticality increase. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2952010]

Transition to turbulence is one of the important phenomena in fluid dynamics. In some flows, transition arises through the global instability. Kármàn vortex shedding behind a bluff body, Rayleigh–Bénard cell in a cross flow, and self-sustained oscillation in hot jet are well-known examples of global instability. Thus, the bifurcation of global instability has been extensively investigated in theoretical studies.[1–14] Quite many flows having the global instability undergo bifurcation through linear global instability. In this case, when the bifurcation parameters (e.g., the Reynolds number and velocity ratio for mixing layer) exceed critical values, disturbances grow exponentially due to unstable linear global mode. When the disturbances reach finite amplitude, the growth rate is attenuated by stabilizing nonlinearity. Finally, the disturbances are saturated to a specific self-sustained nonlinear structure called the nonlinear global mode.

The dynamics of nonlinear global mode belongs to weakly or fully nonlinear regime according to the size of departure of bifurcation parameters from the criticality.[9,14] Within small departure from the criticality, conventional weakly nonlinear theory is applicable to analyze the global instability. The region where the weakly nonlinear theory is valid is called the weakly nonlinear regime but is narrow. In this regime, nonlinear global mode strongly depends on the dynamics of linear global mode and its spatial structure is almost same as that of linear global mode. On the other hand, when the departure from the criticality becomes sufficiently large, weakly nonlinear theory is no more valid and the dynamics of nonlinear global mode is dominated by strong nonlinearity. This region is called the fully nonlinear regime. In this regime, the formation of nonlinear global mode is closely related to the front propagation.[4–14] When the disturbance is triggered by linear global instability, it nucleates a front propagating into upstream. Then, the properties of this propagating front at the trailing and leading edges of a nonlinear wavepacket are selected according to linear criteria.[7,13,14] When this front once stops at a location where its velocity vanishes, it acts like a wavemaker generating instability wave(s) downstream. Finally, the front and instability wave(s) generated form a nonlinear global mode.

Recently, the sensitivity of global instability that sheds light on new aspects of global-instability dynamics has been investigated in a few studies.[14–16] All these studies were conducted in the weakly nonlinear regime using linear global mode, where the sensitivity of global instability was measured by that of the linear global mode and was determined by overlapping region between the regular and adjoint linear global modes. This approach was applied to flow behind a circular cylinder near the onset of Kármàn vortex shedding[15,16] and the results showed good agreements with the experiment[17] in which the effect of a small secondary cylinder located in the wake was studied. In spite of these successful theoretical analyses in the weakly nonlinear regime, the global instability belongs mostly to the fully nonlinear regime. However, the sensitivity of global instability in the fully nonlinear regime has not been investigated so far.

Therefore, the objective of the present study is to investigate the sensitivity of nonlinear global mode in the fully nonlinear regime and compare it to the sensitivity of linear global mode. For this purpose, we consider the supercritical real Ginzburg–Landau equation that is well established as a simplified model in understanding the instabilities of open shear flows such as the wake behind a bluff body,[14]

$$\frac{\partial A}{\partial t} = N(A) + \epsilon f \delta(x - x_f) + \epsilon c(x) A, \tag{1a}$$

where

$$N(A) \equiv -U\frac{\partial A}{\partial x} + \mu(x)A + \frac{\partial^2 A}{\partial x^2} - A^3, \tag{1b}$$

with the boundary conditions of $A(x=0)=A(x=\infty)=0$. Here, $x \in [0,\infty)$ is the streamwise direction, $t \in [0,\infty)$ the time, $A(x,t) \in \mathbb{R}$ the amplitude of instability wave, $U \in \mathbb{R}$ the ad-

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vection velocity, and $\mu(x)$ the control parameter, respectively. $\mu(x)$ is set to linearly depend on x, $\mu(x)=\mu_0-\mu_1 x$, where μ_0 controls the bifurcation of Eqs. (1a) and (1b) and $\mu_1(>0)$ represents a spatial nonparallelism of flow. In Eqs. (1a) and (1b), two types of small perturbations are included to investigate the sensitivity of global instability: $\epsilon f\delta(x-x_f)$ is an open-loop perturbation provided at $x=x_f$, and $\epsilon c(x)A$ is a closed-loop perturbation. The global instability of Eqs. (1a) and (1b) occurs at $\mu_0=\mu_c(\equiv U^2/4+|\zeta_1|\mu_1^{2/3})$, where ζ_1 is the first zero of Airy function ($\zeta_1=-2.338$).[9] For $\mu_0<\mu_c$, Eqs. (1a) and (1b) are globally stable and have unique stable solution $A(x,t)=0$ (basic state), whereas for $\mu_0>\mu_c$ Eqs. (1a) and (1b) become globally unstable and have a steady nontrivial solution $A(x,t)=A_{\mathrm{NG}}(x)$ called the nonlinear global mode (bifurcated state).[9]

Let us discuss how to measure of the sensitivity of global instability. In Refs. 14–16, the sensitivity of global instability is described by the response of the linearized equation about the basic state and the sensitivity of linear global frequency. However, they are inadequate for the study of global instability in the fully nonlinear regime because the linear and weakly nonlinear theories based on linear global mode are not valid in this regime. Therefore, we introduce a quantitative measure to evaluate the sensitivity of global instability for both weakly and fully nonlinear regimes: the global-mode energy, $E_{\mathrm{NG}}=\int_0^\infty A_{\mathrm{NG}}^2(x)dx$, representing the energy of saturated steady solution. To describe the effect of small perturbation on the global-mode energy, we introduce the sensitivity of global-mode energy,

$$\delta E_{\mathrm{NG}}(w)=\lim_{\epsilon\to 0}\frac{E_{\mathrm{NG}}(\epsilon w)-E_{\mathrm{NG}}(0)}{\epsilon}, \tag{2}$$

where $E_{\mathrm{NG}}(\epsilon w)$ is the global-mode energy perturbed by the disturbances $\epsilon w(=\epsilon f\delta+\epsilon cA)$ in Eq. (1a) and $E_{\mathrm{NG}}(0)$ is the global-mode energy without disturbances.

We first study the sensitivity of global instability of Eqs. (1a) and (1b) in the weakly nonlinear regime. Using a standard multiple-scale analysis,[18] for very small departure from the criticality ($\mu_0-\mu_c=\epsilon\Delta\mu_0$), the amplitude of instability wave is expanded about $A=0$ as $A(x,t)=\Sigma_{n=1}^{\infty}\epsilon^{n/2}\rho_n(x,t,T)$ with an assumption that time evolution is very slow ($T=\epsilon t$) and the amplitude of open-loop perturbation is small [$f=\epsilon^{1/2}f_0$; $f_0=O(1)$]. Then, at $O(\epsilon^{1/2})$, this expansion admits the solution $\rho_1(x,t,T)=\kappa(T)\psi_{\mathrm{LG}}(x)e^{-i\omega_G t}$ with $\int\psi_{\mathrm{LG}}^2(x)dx=1$, where $\kappa(T)(>0)\in\mathbb{R}$ is the amplitude of global mode, $\psi_{\mathrm{LG}}(x)\in\mathbb{R}$ is the linear global mode, and ω_{G} is the linear global frequency and is zero at the criticality (i.e., $\mu_0=\mu_c$). At $O(\epsilon)$, the expansion does not provide any information about the leading-order global mode, but at $O(\epsilon^{3/2})$ the solvability condition leads the following Landau equation for $\kappa(T)$:[14]

$$\frac{d\kappa}{dT}=\Delta\mu_0\kappa-\frac{\langle\psi_{\mathrm{LG}}^3,\psi_{\mathrm{LG}}^+\rangle}{\langle\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle}\kappa^3+\frac{f_0\psi_{\mathrm{LG}}^+(x_f)}{\langle\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle}+\frac{\langle c(x)\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle}{\langle\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle}\kappa. \tag{3}$$

Here, a superscript + denotes the adjoint variable and $\langle g,h\rangle$ is the inner product $\int gh$ in $[0,\infty)$.

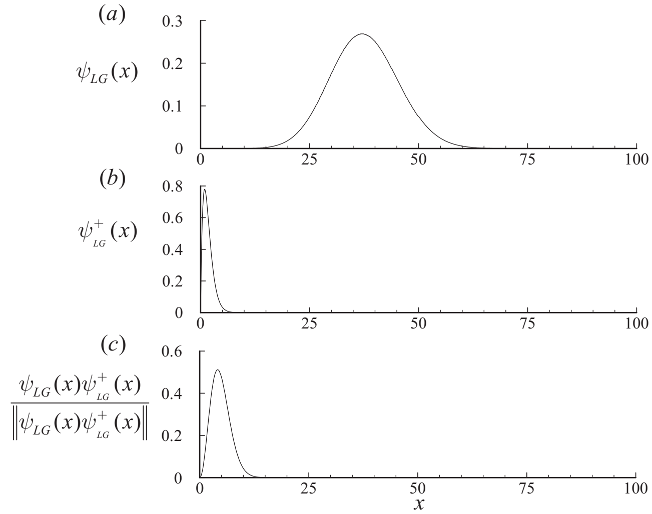

FIG. 1. Linear global mode and its sensitivities with the open- and closed-loop perturbations in the weakly nonlinear regime ($U=2$ and $\mu_1=0.033$): (a) ψ_{LG}; (b) ψ_{LG}^+; (c) $\psi_{\mathrm{LG}}\psi_{\mathrm{LG}}^+/\|\psi_{\mathrm{LG}}\psi_{\mathrm{LG}}^+\|$. Here $\|\cdot\|$ denotes L_2-norm.

The nonlinear global mode A_{NG} in the weakly nonlinear regime is the steady solution of A at $O(\epsilon^{1/2})$: $A_{\mathrm{NG}}(x)=\epsilon^{1/2}\kappa_s\psi_{\mathrm{LG}}(x)$, where κ_s is the nontrivial steady solution of Eq. (3). Thus, $E_{\mathrm{NG}}(\equiv\int A_{\mathrm{NG}}^2 dx)=\epsilon\kappa_s^2$. Then, under the assumption of $\Delta\mu_0\sim O(1)$ and $\langle\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle\gg\langle\psi_{\mathrm{LG}}^3,\psi_{\mathrm{LG}}^+\rangle$ (for example, for $U=2$ and $\mu_1=0.033$, $\langle\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle=7.06\times 10^{-7}$ and $\langle\psi_{\mathrm{LG}}^3,\psi_{\mathrm{LG}}^+\rangle=6.09\times 10^{-15}$), the sensitivities of global-mode energy with perturbations $\epsilon f\delta$ and ϵcA become

$$\delta E_{\mathrm{NG}}(f\delta)\simeq \upsilon^{-2/3}f_0\psi_{\mathrm{LG}}^+(x_f)^{2/3}, \tag{4a}$$

$$\delta E_{\mathrm{NG}}(cA)=\upsilon^{-1}\langle c(x)\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle, \tag{4b}$$

where $\upsilon=\langle\psi_{\mathrm{LG}}^3,\psi_{\mathrm{LG}}^+\rangle$. From Eqs. (4a) and (4b), the sensitivities in the weakly nonlinear regime are measured by the adjoint linear global mode ψ_{LG}^+ and the overlapping region between regular and adjoint modes $\psi_{\mathrm{LG}}\psi_{\mathrm{LG}}^+$, respectively. As the sensitivity of linear global frequency is given as $\langle c(x)\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle/\langle\psi_{\mathrm{LG}},\psi_{\mathrm{LG}}^+\rangle$ in Ref. 14, the sensitivity of global-mode energy is directly related to that of linear global frequency [see Eq. (4b)]. When the linearized equation is strongly non-normal, υ becomes vanishingly small. Then, the coefficient of cubic nonlinear term in Eq. (3) (called the Landau constant) also becomes very small because of vanishingly small υ. In this case, the nonlinear term cannot sufficiently saturate the global mode and the bifurcation becomes very steep. As a result, the region where the weakly nonlinear theory is valid becomes vanishingly small.[14] In addition, the global instability also becomes extremely sensitive to the perturbation because the sensitivities in Eqs. (4a) and (4b) are inversely proportional to $\upsilon^{2/3}$ and υ, respectively.

Figure 1 shows the linear global mode $\psi_{\mathrm{LG}}(=\phi_{\mathrm{LG}}/\int\phi_{\mathrm{LG}}^2dx)$, adjoint linear global mode $\psi_{\mathrm{LG}}^+(=\phi_{\mathrm{LG}}^+/\int\phi_{\mathrm{LG}}^{+2}dx)$, and their overlapping region $\psi_{\mathrm{LG}}\psi_{\mathrm{LG}}^+$ in the weakly nonlinear regime, where $\phi_{\mathrm{LG}}=e^{U/2x}\,\mathrm{Ai}(\mu_1^{1/3}x+\zeta_1)$, $\phi_{\mathrm{LG}}^+=e^{-U/2x}\,\mathrm{Ai}(\mu_1^{1/3}x+\zeta_1)$, and $\mathrm{Ai}(x)$ denotes the Airy function.[1,9] In the weakly nonlinear regime, the global mode $\psi_{\mathrm{LG}}(x)$ evolves sufficiently far downstream [Fig. 1(a)]. On

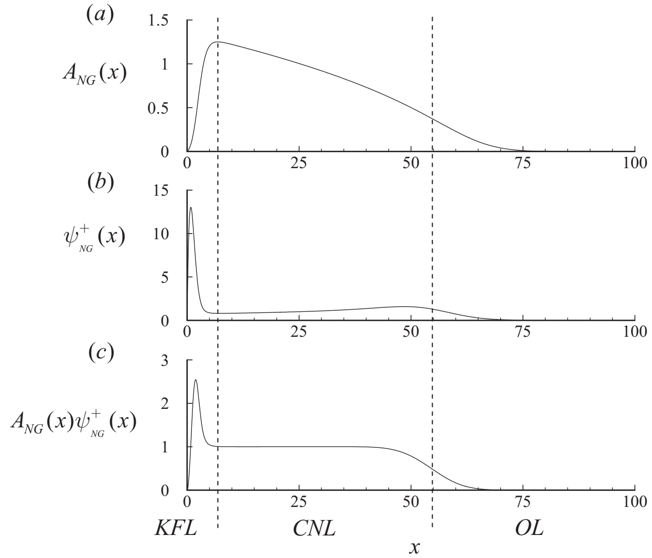

FIG. 2. Nonlinear global mode and its sensitivities with the open- and closed-loop perturbations in the fully nonlinear regime ($U=2$, $\mu_0=1.8$, and $\mu_1=0.033$) (a) A_{NG}; (b) ψ^+_{NG}; (c) $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$. Here, KFL denotes the Kolmogorov front layer, CNL the central nonlinear layer, and OL the outer layer, respectively.

the other hand, the adjoint global mode ψ^+_{LG} representing the sensitivity to open-loop perturbation $\epsilon f\delta$ is concentrated right after the inlet $(0<x<5)$ [Fig. 1(b)]. Thus, the overlapping region $\psi_{\mathrm{LG}}\psi^+_{\mathrm{LG}}$ occurs at $0<x<15$ [Fig. 1(c)], indicating that this region is the most sensitive to the closed-loop perturbation $\epsilon c(x)A$.

Now, we investigate the sensitivity of global instability in the fully nonlinear regime (i.e., $\mu_0-\mu_c \gg \epsilon\Delta\mu_0$). First, we perform numerical simulation of Eqs. (1a) and (1b) to obtain nonlinear global mode $A_{\mathrm{NG}}(x)$. Second-order central difference and fourth-order Runge–Kutta methods are used for spatial discretization and time integration, respectively. The computation domain is $0\leqslant x\leqslant 150$ with 15001 uniformly spaced grid points and the Neumann condition (i.e., $dA/dx=0$) is applied at $x=150$. A_{NG} is obtained after $A(x,t)$ becomes steady and is shown in Fig. 2(a). According to Ref. 9, $A_{\mathrm{NG}}(x)$ consists of three main layers. The layer located right after the inlet is called the Kolmogorov front layer (KFL) that is known to act as a wavemaker of global instability. In this layer, $A_{\mathrm{NG}}(x)$ mainly consists of stationary Kolmogorov front. The layer located right after KFL is the central nonlinear layer (CNL), where $A_{\mathrm{NG}}(x)\sim\sqrt{\mu(x)}$. Finally, the tail of $A_{\mathrm{NG}}(x)$ is called the outer layer (OL). For more details, see Ref. 9.

The amplitude of instability wave perturbed by $\epsilon f\delta(x-x_f)$ and ϵcA is expanded about $A=A_{\mathrm{NG}}$ as $A(x)=A_{\mathrm{NG}}(x)+\epsilon\psi_{\mathrm{NG}}(x)+O(\epsilon^2)$. Then, we obtain the following linearized equation for $\psi_{\mathrm{NG}}(x)$ (called the secondary-instability equation):

$$\left.\frac{\mathcal{D}N(A)}{\mathcal{D}A}\right|_{A_{\mathrm{NG}}}\psi_{\mathrm{NG}}+f\delta(x-x_f)+c(x)A_{\mathrm{NG}}(x)=0, \qquad (5)$$

where $(\mathcal{D}N(A)/\mathcal{D}A)|_{A_{\mathrm{NG}}}$ is the linearized operator about A_{NG} and $\psi_{\mathrm{NG}}(0)=\psi_{\mathrm{NG}}(\infty)=0$. Using the method of variational calculus, δE_{NG} is obtained as

$$\delta E_{\mathrm{NG}}(f\delta)=f\psi^+_{\mathrm{NG}}(x_f), \qquad (6a)$$

$$\delta E_{\mathrm{NG}}(cA)=\langle c(x)A_{\mathrm{NG}},\psi^+_{\mathrm{NG}}\rangle, \qquad (6b)$$

where

$$\left.\frac{\mathcal{D}N(A)}{\mathcal{D}A}\right|^{+}_{A_{\mathrm{NG}}}\psi^+_{\mathrm{NG}}(x)+2A_{\mathrm{NG}}(x)=0, \qquad (6c)$$

and $\psi^+_{\mathrm{NG}}(0)=\psi^+_{\mathrm{NG}}(\infty)=0$. It is interesting to note that Eqs. (6a) and (6b) are analogous to Eqs. (4a) and (4b), respectively, even though the method of evaluating the sensitivities of global-mode energy in the fully nonlinear regime is essentially different from that in the weakly nonlinear regime. Thus, similar to Eqs. (4a) and (4b), the sensitivities of global-mode energy with $\epsilon f\delta(x-x_f)$ and $\epsilon c(x)A$ are measured by the adjoint mode of secondary-instability equation ψ^+_{NG} and the overlapping region between the nonlinear global and adjoint modes $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$, respectively.

Figures 2(b) and 2(c) show ψ^+_{NG} and $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$, respectively. ψ^+_{NG}, representing the sensitivity to open-loop perturbation $\epsilon f\delta$, is large right after the inlet $(0<x<3)$ [Fig. 2(b)], and $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$ has large values in $0<x<5$ [Fig. 2(c)]. Note that the maxima of ψ^+_{NG} and $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$ are located in KFL, indicating that KFL is responsible for the generation of global mode. The present result qualitatively agrees with those by Refs. 10 and 11 in which nonlinear Wentzel–Kramers–Brillouin–Jeffreys theory is used to show that the Kolmogorov front plays the role of wavemaker. It is also noteworthy that both ψ^+_{NG} and $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$ are nearly constant in CNL and have appreciable amplitudes unlike the linear global modes, implying that the sensitivity is more spread out in the fully nonlinear regime than it is in the weakly nonlinear regime, particularly for closed-loop perturbations.

Now, we investigate the sensitivities of linear and nonlinear global modes with respect to the departure from the criticality $\delta_0=(\mu_0-\mu_c)/\mu_c$. Figures 3(a) and 3(b) show maxima of the sensitivities in the weakly and fully nonlinear regimes, respectively. In the weakly nonlinear regime [Fig. 3(a)], maxima of $v^{-2/3}\psi^{+2/3}_{\mathrm{LG}}$ and $v^{-1}\psi_{\mathrm{LG}}\psi^+_{\mathrm{LG}}$ do not change with δ_0 because ψ_{LG} and ψ^+_{LG} are independent of μ_0. On the other hand, in the fully nonlinear regime [Fig. 3(b)], both maxima of ψ^+_{NG} and $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$ decrease with increasing δ_0. This result indicates that the nonlinear global mode in the fully nonlinear regime becomes less sensitive to the perturbation with increasing δ_0. Thus, the behavior of the sensitivity of nonlinear global mode in the fully nonlinear regime is very different from that of linear global mode. On the other hand, as shown in Figs. 1 and 2, the locations of maxima of ψ^+_{LG} and $\psi_{\mathrm{LG}}\psi^+_{\mathrm{LG}}$ are similar to those of ψ^+_{NG} and $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$, indicating that the sensitive regions of linear global mode to disturbances are analogous to those of nonlinear global mode even in the fully nonlinear regime.

The variations of nonlinear global mode and its sensitivities with the departure from the criticality δ_0 in the fully nonlinear regime are shown in Fig. 4. As δ_0 increases, A_{NG} increases and its maximum location moves upstream. On the other hand, maxima of both ψ^+_{NG} and $A_{\mathrm{NG}}\psi^+_{\mathrm{NG}}$ located in KFL rapidly decay with increasing δ_0 [see also Fig. 3(b)]. This

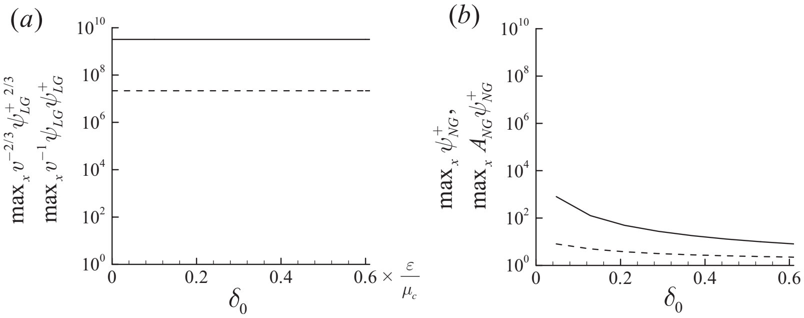

FIG. 3. Maxima of the sensitivities of global-mode energy with respect to the departure from the criticality δ_0 (U=2 and μ_1=0.033). (a) Weakly nonlinear regime (linear global mode): (—) $\max_x \upsilon^{-2/3}\psi_{\mathrm{LG}}^{+\,2/3}$; (---) $\max_x \upsilon^{-1}\psi_{\mathrm{LG}}\psi_{\mathrm{LG}}^{+}$; (b) fully nonlinear regime (nonlinear global mode): (—) $\max_x \psi_{\mathrm{NG}}^{+}$; (---) $\max_x A_{\mathrm{NG}}\psi_{\mathrm{NG}}^{+}$.

indicates that the role of Kolmogorov front as a wavemaker becomes less important when the departure is sufficiently far from the criticality. In CNL, ψ_{NG}^{+} decreases with increasing δ_0 but $A_{\mathrm{NG}}\psi_{\mathrm{NG}}^{+}$ is nearly constant, indicating that CNL is almost insensitive to the variation of δ_0 for the closed-loop perturbation. With increasing δ_0, the tails of both ψ_{NG}^{+} and $A_{\mathrm{NG}}\psi_{\mathrm{NG}}^{+}$ move downstream and the width of CNL increases.

So far, the results from fixed values of the advection velocity U and the spatial nonparallelism μ_1 were given in this letter. Certainly, these parameters influence the nonnormality of linearized operators about both the basic and bifurcated states and thus the characteristics of linear and nonlinear global modes. Therefore, we tested a few different values of U and μ_1 but the main findings described above did not change.

In summary, we have investigated the sensitivity of global instability in the weakly and fully nonlinear regimes. The main conclusions from the present study are such that in the fully nonlinear regime, (1) the sensitivity of global instability is determined by nonlinear global mode and adjoint mode of secondary-instability equation; (2) the Kolmogorov front acts as a wavemaker of nonlinear global mode; (3) the sensitive regions of nonlinear global mode are similar to those of linear global mode in the weakly nonlinear regime; (4) the role of Kolmogorov front as a wavemaker becomes less important for sufficiently large departure from the criticality. Although the present results are obtained from a simplified model equation, we hope that the present study will shed light on the behavior of flows such as the instability in the wake of a bluff body.

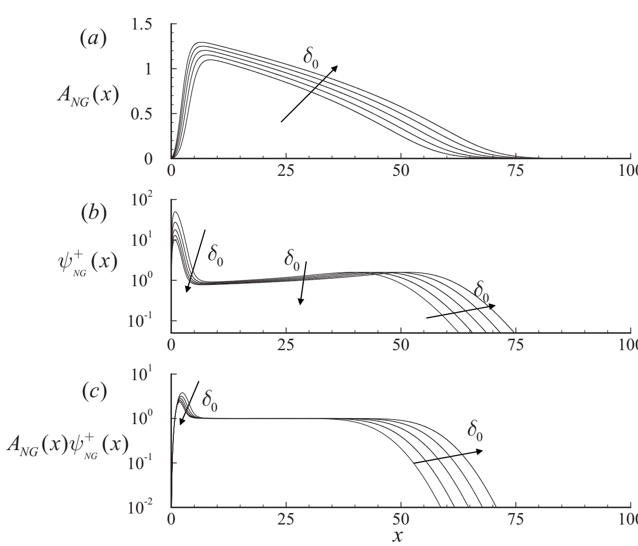

FIG. 4. Variations of nonlinear global mode and its sensitivities with the departure from the criticality δ_0 in the fully nonlinear regime (U=2 and μ_1=0.033): (a) A_{NG}; (b) ψ_{NG}^{+}; (c) $A_{\mathrm{NG}}\psi_{\mathrm{NG}}^{+}$. Here, δ_0's are from 0.21 to 0.53 by increments of 0.08.

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