Worksheet for resolvent methods

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You have been given a short Matlab function to calculate the Orr-Sommerfeld-Squire operator for a channel. To keep things simple the operator is formed around the laminar profile. In the formulation used in the code, the velocity field is Fourier transformed in space, so has streamwise wavenumber k_x and spanwise wavenumber k_z,

$$\mathbf{u}(x, y, z, t) \propto \int_{k_x} \int_{k_z} \hat{\hat{\mathbf{u}}}(y, t; k_x, k_z) \, dk_x dk_z.$$

As such, a Fourier transform in time will give travelling waves with downstream streamwise wavespeed $-\omega/k_x$.

Exercise 0.1. Examine the function `oss.m` and try to understand what it does. Find the function return values A, Q, C and y using resolution $N = 150$, streamwise wavenumber $k_x = 1$, spanwise wavenumber $k_z = 1$, and Reynolds number $Re = 1000$. The matrix A is the discretised Orr-Sommerfeld-Squire operator, Q is the inner product matrix, C allows calculation of the velocity Fourier coefficients at the wall-normal gridpoints from the state x and y is the gridpoints.

Exercise 0.2. Plot the eigenvalue spectrum of A. Look for the eigenvalues closest to the imaginary axis.

Exercise 0.3. Write a function to find the resolvent of A for a given frequency ω. Remember to use the inner product on Q (i.e. the amplitude being given by x^*Qx) for both forcing and response.

Hints:

- You will need the Cholesky decomposition of Q, $Q = W^*W$ (Matlab function `chol`).

- You may find it convenient to define a variable $z = Wx$ such that the energy is calculated simply as z^*z.

Exercise 0.4. Plot the singular values of the resolvent you calculated. Look at how they decay.

Exercise 0.5. Plot the leading singular value as it changes with ω. Compare the values of ω where there is the highest gain to the location of the eigenvalues.

Exercise 0.6. Find the leading response mode at $\omega = -1$. Plot the wall-normal velocity's Fourier coefficient as a function of y.

Exercise 0.7. These exercises are more time consuming. Try them later.

1. Find the leading resolvent modes without explicitly inverting $i\omega - A$
2. Explore the relationship between singular value, wavenumber, frequency and the location of the mode peak
3. Find the same modes using the `eigs` function